
Facilitating Non-Visual Access to Math Equations on Braille Displays

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Abstract

In this paper we describe of two approaches to facilitate the non-visual access to simple math equations on a braille display: abbreviated display and selective display. These approaches intend to alleviate the limitation of the number of cells in braille displays and the difficulty to understand the equation structure by offering reduced or less detailed representations of mathematical equations, taking into consideration the context in which these will be manipulated. We then give some perspectives of implementation into a math editor.

Author Keywords

Non-visual access; visualization; braille; accessibility; mathematics.

ACM Classification Keywords

H.5.m. Information interfaces and presentation (e.g., HCI): Miscellaneous.

Introduction

People with blindness are able to access contents in desktop computers and mobile devices by way of audio and braille. Audio output is possible through text-to-speech applications (also called screen-readers), and braille output through refreshable braille displays (Figure 1). On one hand, audio access is in most cases

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Figure 1: 40-cell braille display (Braille Edge 40, from HIMS)

	1	2	3
Français			
CMU			
Nemeth			

Figure 2. Representation of numbers on different braille math codes. As opposed to literary braille, braille mathematical codes are not unified; for example in Spain and Latin American countries they use the *Código Matemático Unificado (CMU)*, in the United States the *Nemeth* code, in France the *Notation mathématique braille*, in Germany and Austria the *Marburg* code.

cost-free since it is integrated in the device's operating system, but it is slower compared to visual access. On the other hand, most braille displays are expensive but its tactile access is faster than audio. A braille display can be connected to a desktop computer or mobile device and its output is synchronized with the screen reader's output; some braille devices include a keyboard for input and therefore can be used standalone. These devices are very practical for braille users, but they have a limited number of characters that can be displayed in one line, which can range from 14 to 80 characters. When displaying literary contents, the length of the braille display is not a critical limitation, since the order of reading is linear and when the line is interrupted the idea continues immediately after. On the other hand, reading mathematical contents in a linear display is different, since in order to comprehend the syntactical structure of a mathematical expression it is necessary to view it as a whole, and then to analyze its parts or sub-expressions. Besides, in order to make transformations to an expression the reader needs to access different terms, which are not necessarily in sequential order. In comparison with the mathematical notation in black which has a specific spatial arrangement (i.e. exponents, numerators, denominators, radicands are located in a specific relative position to other terms), the different mathematical braille codes (Figure 2) are linear. There exist several math editors which offer edition of expressions either in linear or spatial arrangement; in both cases the non-visual access is through audio and braille, which have limitations that need addressing.

Linear vs spatial arrangement

It is well known that the spatial arrangement of mathematical operations has an impact in the ease of

reading and solving [1][6][7]. Reading a mathematical expression rendered linearly is slower than reading an expression rendered spatially. In order to have an idea of the difference, try locating the denominators in the linear representation:

$$(64x^2 - 32x + 36x - 18)/36 - (16x^2 + 24x + 9)/9 - (x - 1)/9 = 1$$

Now do the same in the spatial representation:

$$\frac{64x^2 - 32x + 36x - 18}{36} - \frac{16x^2 + 24x + 9}{9} - \frac{x - 1}{9} = 1$$

In the second representation, denominators stand out thanks to the visual arrangement of the equation; this is a possibility that users of a linear representation, such as users with blindness using screen-readers and braille displays, do not have. Even using a tactile access with a braille display, the user will take some time to locate the terms of reference. In the same way that a user with sight can easily identify the terms of an expression, users with blindness require to have an equivalent possibility.

Passive vs active reading

Software applications such as text editors and web browsers have tackled the issue of linear access by offering active reading. As opposed to passive reading where the screen-reader speaks aloud the contents of a document in linear sequence and the user listens passively, active reading enables users to have the screen-reader read the contents that he or she wishes to hear. This is made possible by text editors, which provide styles in order to convey the structure of a document (i.e. titles and subtitles), which in turn enable screen-readers -and therefore braille displays- to offer alternatives of navigation, for example

Desired added functionality on a math editor in order to work with basic algebraic operations [3] [4] [5] [9] [10] [11]:

- offer a glance of the equation
- support the comprehension of the syntactical structure of an expression
- find specific positions in the equation
- mark specific positions without having to browse the full equation
- provide a non-sequential access to the terms in the expression.

navigating through first level headings without having to listen or show the contents of second level headings. However, most software applications that allow the edition and display of mathematical contents do not provide an equivalent functionality; in fact, current screen-readers are not able to correctly read mathematical contents rendered spatially. For example, even a simple term such as x^2 will be read as “x two” by most screen-readers.

Active reading of equations is now possible in some of the applications and prototypes aimed in particular, but not exclusively, to users with blindness. LAMBDA [8] uses a linear notation, and Infy [13] and the prototype Algebra Editor [4] use spatial notation; these applications offer active reading by allowing the user to browse a mathematical expression, while using visual and audio output. Some of them also offer braille output on a specific mathematical code. With the aim to improve the capabilities of math editors, we propose some functionalities that could be implemented in order to facilitate non-visual access to contents, with a special focus on their display on braille devices.

Proposal

This proposal was based on an analysis of the tasks involved in the most common operations in basic algebra [10]: multiplication of binomials and polynomials, simplification of terms, operations with fractions, solving systems of linear equations.

We propose the facilitation of non-visual access to an expression from two approaches: abbreviated display and selective display. The display would be offered in three synchronized modes: visual (notation in black), auditive (screen-reader) and tactile (braille display),

considering that the editors that would potentially implement these functions are aimed to both sighted and non-sighted users. In order to illustrate the proposed display types, we will use the following example equation:

$$(64x^2 - 32x + 36x - 18)/36 - (16x^2 + 24x + 9)/9 - (x - 1)/9 = 1$$

This equation occupies roughly 47 braille cells, with a small variation depending on the math braille code. In a 40-cell braille display this equation would take two lines, which the user would read one by one; in a 20-cell display it would take three lines.

Abbreviated display

This approach covers partially the need for a glance of the expression, and offers support in the comprehension of the syntactic structure of the equation. When reading or working with a mathematical equation, users require to understand the syntactic structure of the equation in its current form, in order to prepare a solving strategy [2]. For example, if the task is to solve the example equation the user would have to identify first that it is involved an operation with fractions, in this case an addition/subtraction. The abbreviated display is based on a proposal by [5], and it consists of showing a shorter version of the full equation, by previously folding its sub-expressions into blocks. These blocks would have a short label indicating the type of sub-expression that is folded; i.e. FRAC for fraction, FNUM for numerator, FDEN for denominator, SQRT for square roots, PROD for products, POW for powers, MEM for member, and so on. The user would always be in charge of folding or unfolding the sub-expressions. In the example, if the user chooses to fold the terms to the next level up in the structure, the equation would

Spatially arranged equation

$$\frac{64x^2 - 32x + 36x - 18}{36} - \frac{16x^2 + 24x + 9}{9} - \frac{x - 1}{9} = 1$$

Linearly arranged equation

Approximate view in a 24-cell display (3 lines):

$$(64x^2 - 32x + 36x - 18)/36 - (16x^2 + 24x + 9)/9 - (x - 1)/9 = 1$$

Abbreviated display examples of the equation:

Folding all nodes (1 line in a 24-cell display):

$$FRAC - FRAC - FRAC = 1$$

Unfolding all the terms of the first sub-expression (2 lines):

$$(64x^2 - 32x + 36x - 18)/36 - FRAC - FRAC = 1$$

be displayed as FRAC-FRAC-FRAC=1, using only 16 cells of the display, and having a consequent shorter audio description. Then, the user could choose which block to unfold in order to analyze it, always having control of the sub-expressions he or she wants to display. A related approach of summarization is described in [12], in which informal user feedback of the prototype suggested that short labels are preferred to long ones, and that the expression should be showed fully unfolded so that users would decide how to control its display.

Selective display

Continuing with the previous example, once the user identifies the type of equation he or she is working with, he or she would have to identify the denominators in order to begin the procedure. A user with blindness would have to browse the expression term by term until he or she finds the denominators. Depending on the situation, the user might be looking for denominators, exponents, operators, common terms, etc. Selective display basically consists in displaying exclusively the contents indicated by the user, giving him or her the possibility to toggle between the full and the selected view; in this case, with a previous indication by the user, instead of displaying

$$(64x^2 - 32x + 36x - 18)/36 - (16x^2 + 24x + 9)/9 - (x - 1)/9 = 1$$

the display would show the following terms:

$$\frac{\quad}{9} \quad \frac{\quad}{36} \quad \frac{\quad}{9}$$

which would take considerably less time when read aloud, and which could be very easily found by the user of a braille display.

The concern of this approach is to save time locating specific terms, which are so quickly located in an equation spatially arranged. Observe that the displayed terms appear in their original position, so that the user can keep track of them when he or she toggles to the full view. The selective display does not always represent less lines in the braille display, but less terms shown, which in turn means less verbosity and therefore less cognitive processing.

Another situation that motivated this type of display is the difficulty to multiply binomials, which require locating specific terms in the equation, writing their partial result in the next line, and finding them again quickly when going back to the previous line. In a math editor, an alternative would be to visualize exclusively the relevant terms so they can be located more quickly. For example, in the following expression the user could display only the terms involved in the first set of multiplications:

$$\frac{(3a^2 + 2a + 7)(6a + 4)}{3a^2} \quad \frac{\quad}{(6a + 4)}$$

this would help the user save time locating the $3a^2$ once he comes back from the next line where the partial result is written, or at least it would spare him the effort on keeping track on which term is being multiplied. This approach covers partially the requirements for finding specific positions in the equation without having to browse the full equation, and for providing a non-sequential access to the terms in the expression.

In order to indicate what to display, we propose different search options, which results would be highlighted for selective display. The highlighted terms

Selective display

Displaying *denominators* in the example equation

Approximate view in a 24-cell display (3 lines):

$$\frac{\quad}{9} \quad \frac{36}{9}$$

Approximate view in a 40-cell display (2 lines):

$$\frac{\quad}{9} \quad \frac{36}{9}$$

Displaying *operators* in the example equation

Approximate view in a 24-cell display (2 lines):

$$\left(\begin{array}{ccccc} - & + & - & & \end{array} \right) / \begin{array}{c} - \\ + \end{array} \\ \left(\begin{array}{ccccc} + & + & & & \end{array} \right) / \begin{array}{c} - \\ - \end{array} \right) / =$$

Approximate view in a 40-cell display (2 lines):

$$\begin{array}{ccccc} - & + & - & & \end{array} \right) / \begin{array}{c} - \\ + \end{array} \left(\begin{array}{ccccc} + & + & & & \end{array} \right) / \begin{array}{c} - \\ - \end{array} \right) / =$$

are intended to stay highlighted unless the user indicates otherwise; this would allow the user to make combinations of searches.

Implementation perspectives

We propose to implement the abbreviated and the selective display on the application side; in other words, the editor itself must provide the prepared view so that the braille display can just show it, as opposed to an implementation which would depend on the capabilities of the device. The visual output can be done in Presentation MathML, the corresponding audio output requires preprocessing a textual representation of what will be read by the screen-reader, and the braille output requires an equivalence table containing the ASCII characters that represent each math symbol, in order to convert it to a specific braille math code. A controller module would be in charge of preprocessing the text for the audio and the ASCII braille, and of synchronizing the three output modalities or views.

Conclusion

The abbreviated display and the selective display are alternative views of mathematical expressions, which could help users with blindness to use more effectively the braille display; however, they are yet to be fully implemented and evaluated. If they proved to be effective, they could be used not only for braille display users with limited display space, but also for anyone working with linear mathematical representations, such as LaTeX or LAMBDA, in order to offer a “cleaner” and more succinct view of the contents.

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